$$\sum_{i=1}^{n} \frac{c_i^2 d_i}{(1+\lambda d_i)^2} = 0,$$

which, after multiplying out the denominator, becomes a sixth degree polynomial in λ. This polynomial can be efficiently and accurately solved using standard techniques (the MATLAB function *roots* is used). All solutions are plugged in to determine x, as indicated above, and choose the real solution that minimizes the optimization criteria.

Experiments

Experiments have been performed with the recognition methods of the present invention, using a subset of a data base of faces. This subset contains 3D models of ten faces, including models of their albedos in the red, green and blue color channels. As test images, 42 images of a single individual are used, taken across seven different poses and six different lighting conditions (shown in Figure 4). In these experiments, each image is compared to each model, and the rank of the correct answer is determined (i.e., a rank of one means the correct answer was chosen first). This subset of the data base is too small to allow us to draw any definite conclusions from the experiments. Rather, it is small enough to allow us to compare a number of different methods, some of which are too slow to run on a large data set.

In implementing all the methods, one must first obtain a 3D alignment between the model and the image (referred to as "positioning"). This can be done with existing methods known in the art, such as with the method disclosed in co-pending U.S. Patent Application, Serial Number 09/538209 (Attorney Docket Number NECI 1443, 13414), which is incorporated herein by its reference. In brief, features on the faces can be identified by hand, and then a 3D rigid transformation can be found to align the 3D features with the corresponding 2D image features. For instance, suppose the 3D models are models of many people's faces. Prior to recognition, a person can click on points on the face, indicating the location of features like the center of the eyes or the tip of the nose. When the input image arrives, a person can click on corresponding features in the input image. Given a match between the image features and the model features, for each model it can be determined the position of that object relative to the camera that best matches the model features to the input image features. Determining this positioning is a well-studied problem in the art, for which many solutions have been derived.